Solving Trigonometric Equations

Solve the trigonometric equation below and give only the solutions that lie in the interval $0 \leq x < 2\pi$.

$$\sqrt{3} - cos(-x) = cos\, x$$

$\sqrt{3} - \cos x = \cos x$

$2 \cos x = \sqrt{3}$, $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$ or $x = \frac{11\pi}{6}$

cosine is positive in the first + fourth quadrants.

Solve the trigonometric equation below expressing all solutions using radian measure.

$$\sqrt{3} - cos(-x) = cos\, x$$

$\cos x = \frac{\sqrt{3}}{2}$, $x = \frac{\pi}{6} \pm 2k\pi$,

$x = \frac{11\pi}{6} \pm 2k\pi$, $k = 0, 1, 2, 3, \ldots$

Strategies for Solving a Trigonometric Expression.

1. Add or subtract a quantity to both sides of the equation.
2. Factor an expression.
3. Multiply by a common denominator.
4. Take the square root of both sides of an equation.
5. Square both sides of an equation.
6. Use identities.
7. Use inverse trig functions*

**Be careful to consider the range of the inverse trig functions.*

Solve the trigonometric equation below expressing all solutions using radian measure.

$4\,csc^2 x + 3 = 19$

$4 \csc^2 x = 16$, $\csc^2 x = 4$, $\dfrac{1}{\sin^2 x} = 4$

$\sin^2 x = \frac{1}{4}$, $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} \pm 2k\pi$, $k = 1, 2, 3, \ldots$

Solve the trigonometric equation below and express all solutions that lie in the interval $[0, 2\pi)$.

$sec^2 x - 7sec\, x + 12 = 0$

Let $w = \sec x$.

$w^2 - 7w + 12 = 0$

$(w-4)(w-3) = 0$, $w = 4$ or $w = 3$

$\sec x = 4$ or $\sec x = 3$

$\cos x = \frac{1}{4}$ or $\cos x = \frac{1}{3}$

$x = \cos^{-1}\frac{1}{4}$ or $x = 2\pi - \cos^{-1}\frac{1}{4}$

$x = 1.318$ or $x = 4.965$

or

$x = \cos^{-1}\frac{1}{3}$ or $x = 2\pi - \cos^{-1}\frac{1}{3}$

$x = 1.231$ or $x = 5.052$

Solve the trigonometric equation below and express all solutions that lie in the interval $[0^o, 360^o)$.

$sin^2\theta + \frac{3}{2} sin\,\theta + 1 = \frac{5}{2} - sin\,\theta$

$w = \sin\theta \qquad w^2 + \frac{3}{2}w + 1 = \frac{5}{2} - w$

$2w^2 + 3w + 2 = 5 - 2w$

$2w^2 + 5w - 3 = 0$

$(2w - 1)(w + 3) = 0$

$w = \frac{1}{2}$ or $w = -3$

$\sin\theta = \frac{1}{2}$ or $\underbrace{\sin\theta = -3}_{\text{no solutions}}$

$\theta = 30^0$

or

$\theta = 150^0$

($\sin\theta$ is positive in the first & fourth quadrants)

Solve the trigonometric equation below expressing all solutions using radian measure.

$\sqrt{3}\,cos(2x) - sin(2x) = 0$

$$\sin(2x) = \sqrt{3}\cos(2x)$$

$$\frac{\sin(2x)}{\cos(2x)} = \sqrt{3}, \quad \tan(2x) = \sqrt{3}$$

$$2x = \frac{\pi}{3} \quad \text{or} \quad 2x = \pi + \frac{\pi}{3}$$

(tangent is positive in the first + third quadrants)

$$\text{so} \quad 2x = \frac{\pi}{3} \pm 2k\pi$$

$$x = \frac{\pi}{6} \pm k\pi, \quad k = 0, 1, 2, 3, \ldots$$

or

$$2x = \pi + \frac{\pi}{3} \pm 2k\pi = \frac{4\pi}{3} \pm 2k\pi$$

$$x = \frac{2\pi}{3} \pm k\pi, \quad k = 0, 1, 2, 3, \ldots$$